

# Adjoint-related activities at Harvard

Jake Gebbie, Harvard

1. Estimating the steady-state ocean circulation
2. Projects with the TAF-produced MOM4 adjoint model

(Gebbie, Tziperman, Lee and Fukumori)



## MOM4\_ad



JPL



FastOpt

# A constraint for steady-state ocean circulation

A conservative tracer in steady-state can have no internal extrema.

Conservation of tracers and mass

$$\theta = \sum_{i=1}^N \theta^{(i)} m_i + \eta_\theta$$

$$S = \sum_{i=1}^N S^{(i)} m_i + \eta_s$$

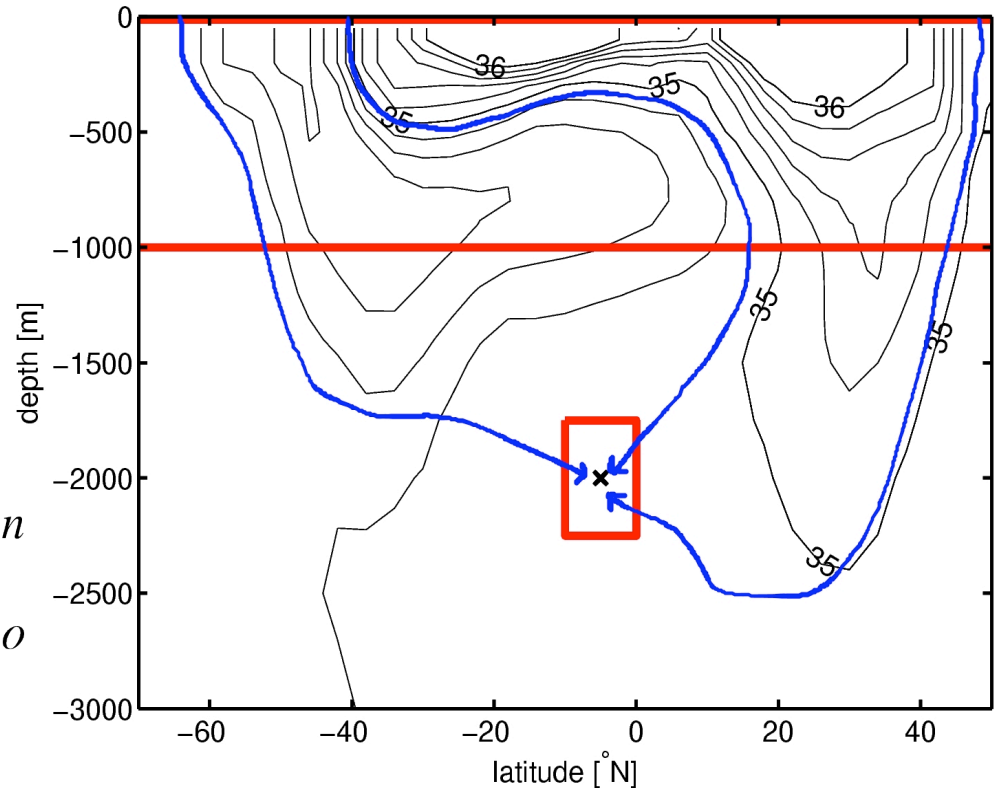
$$\delta^{18}\text{O} = \sum_{i=1}^N \delta^{18}\text{O}^{(i)} m_i + \eta_o$$

$$\text{PO}_4 = \sum_{i=1}^N \text{PO}_4^{(i)} m_i + \Delta P + \eta_p$$

$$\text{NO}_3 = \sum_{i=1}^N \text{NO}_3^{(i)} m_i + 16\Delta P + \eta_n$$

$$\text{O}_2 = \sum_{i=1}^N \text{O}_2^{(i)} m_i - 175\Delta P + \eta_o$$

$$1 = \sum_{i=1}^N m_i$$



The steady-state constraint is symbolically represented as  $F(x) = 0$ , where  $x$  includes the tracer fields, and all “ $m$ ” variables can vary between 0 and 1. Data from Gouretski & Koltermann (WOCE)

Minimize

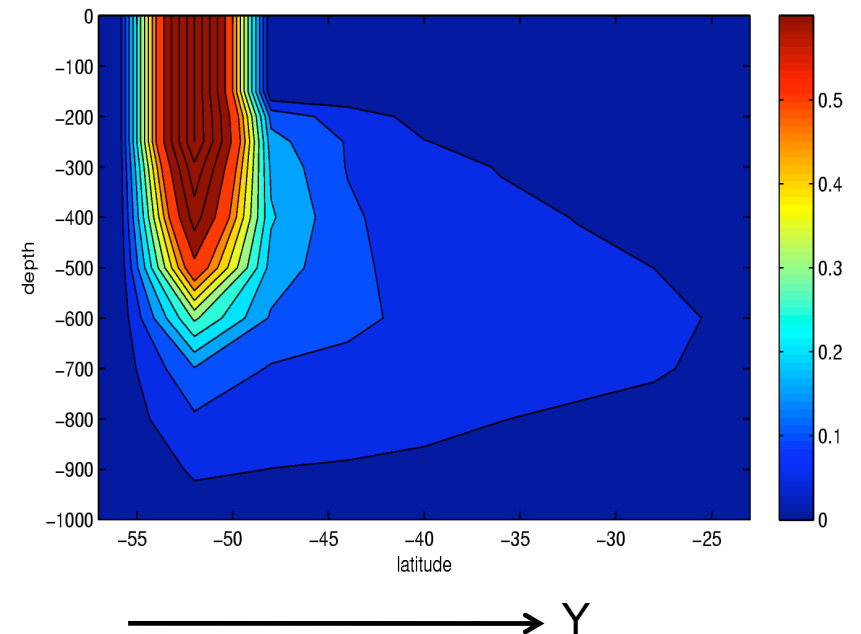
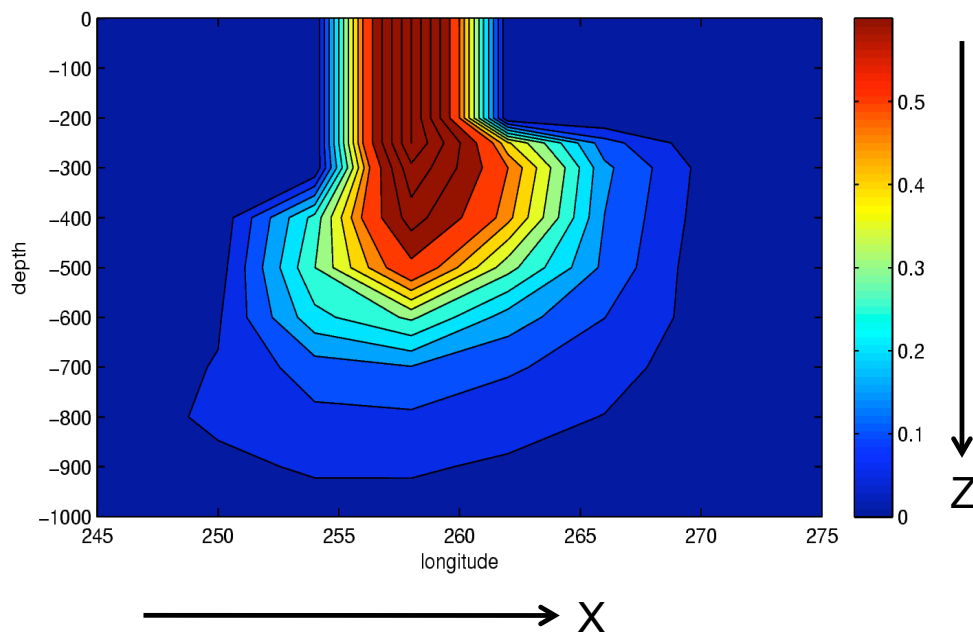
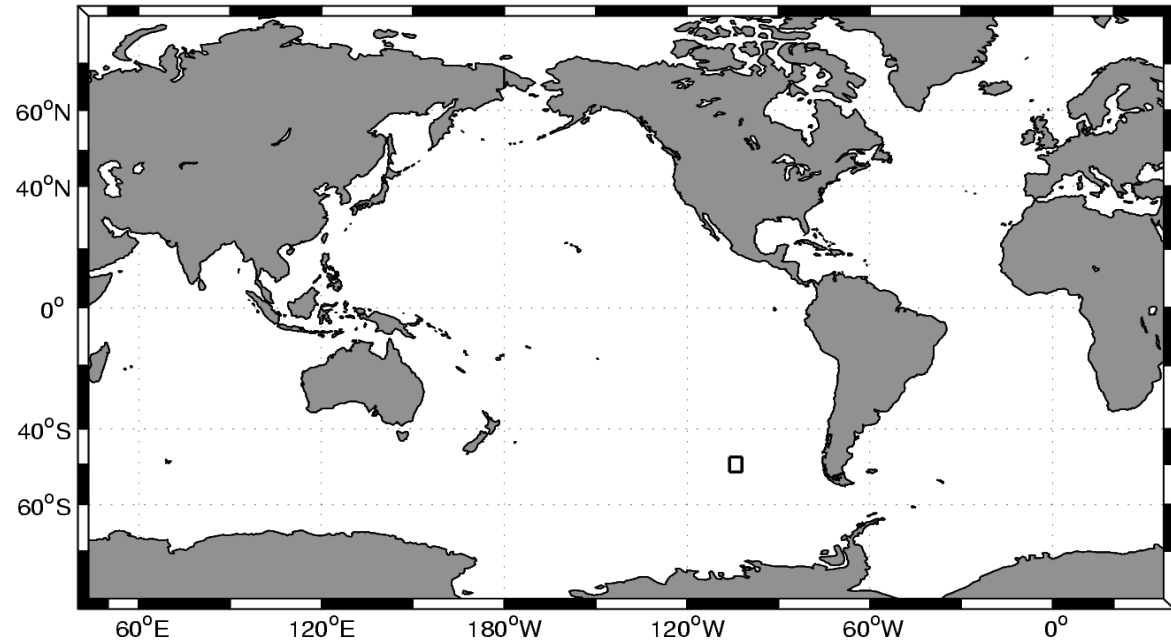
$$J = (x - x_{\text{obs}})^T W (x - x_{\text{obs}})$$

subject to  $F(x) = 0$

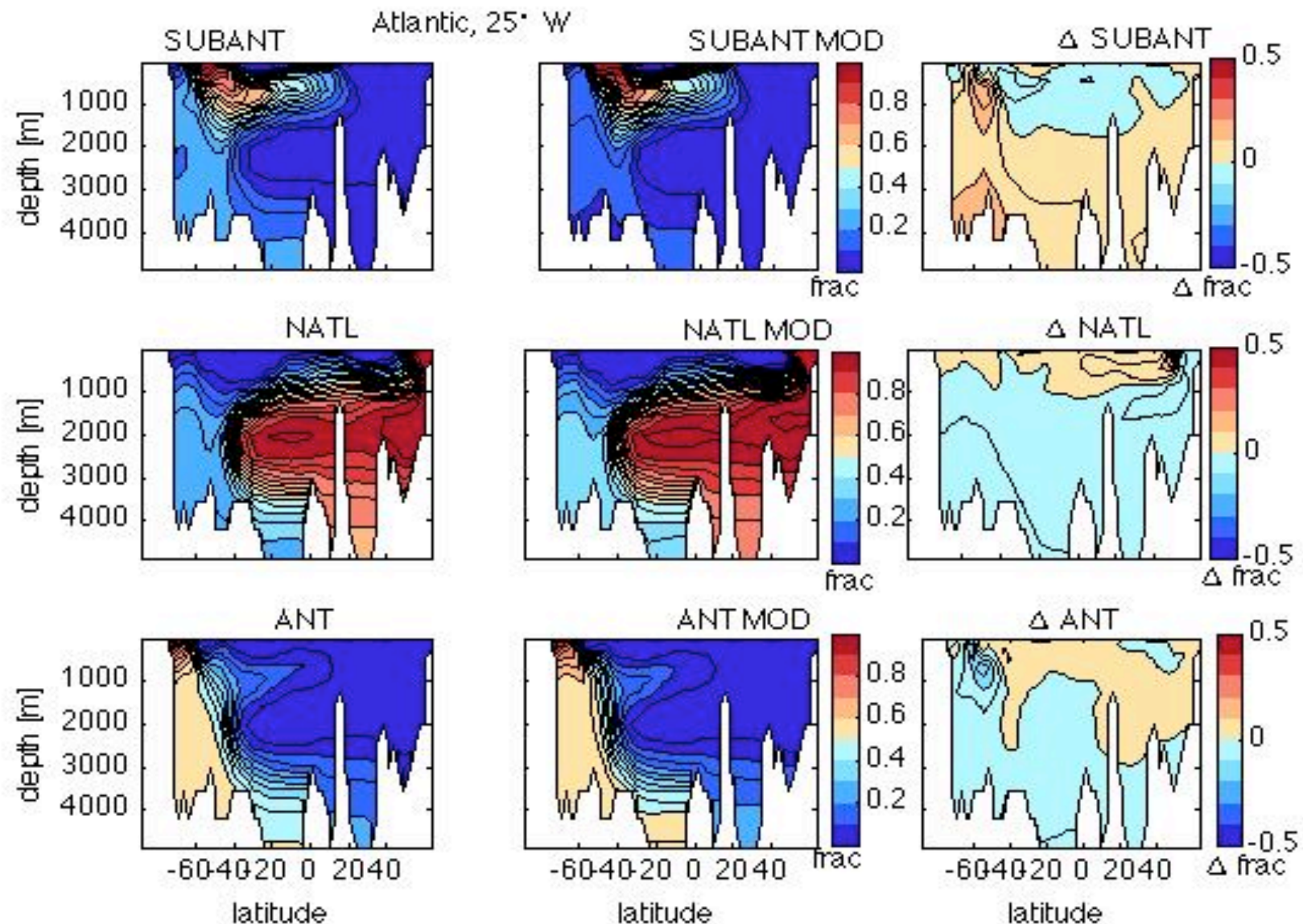
where

$$x = [T, S, P, N, O, O18]$$

A decomposition  
into 9,337 distinct  
water types

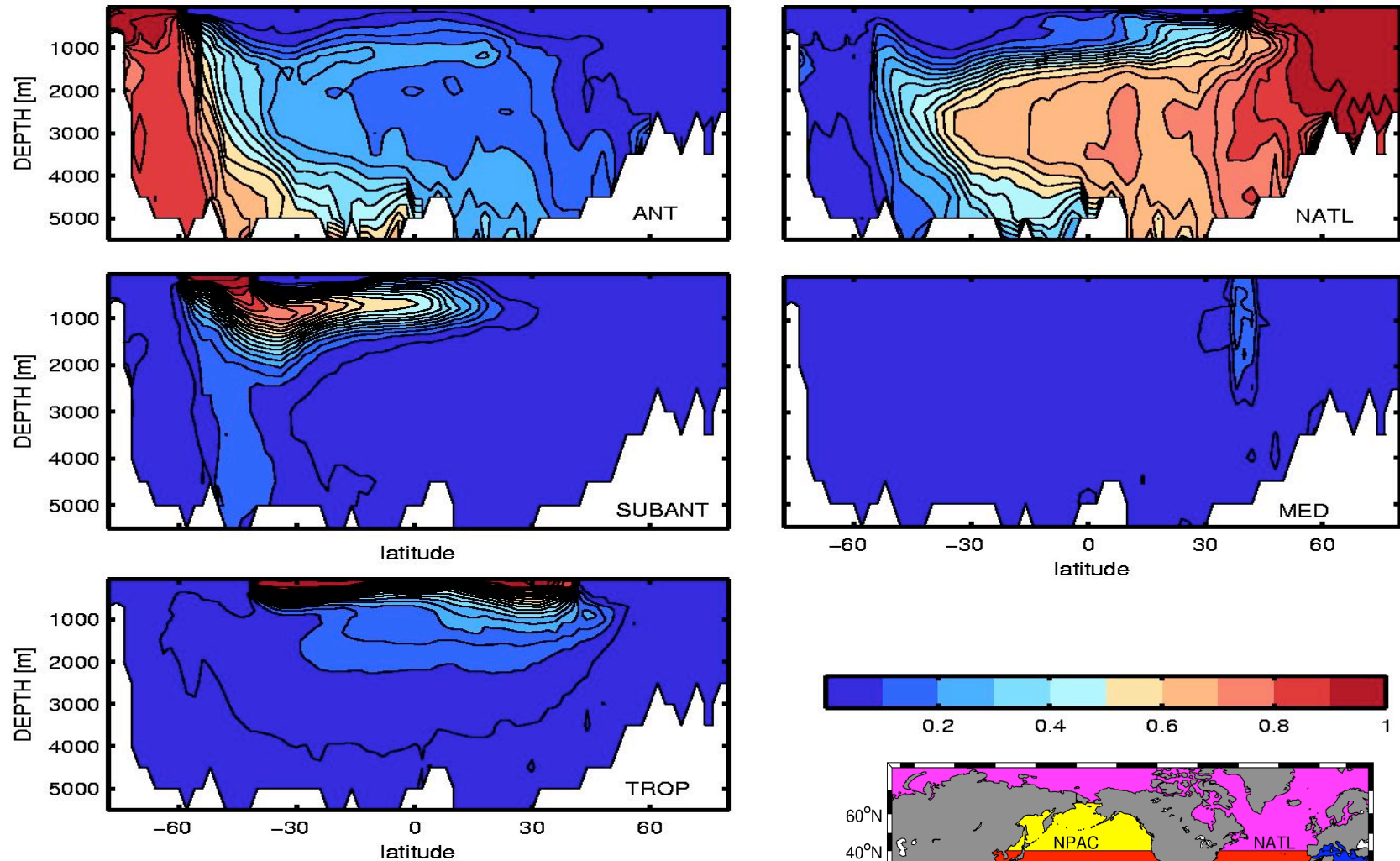


# Volume fraction from regional sources (Identical twin)



Gebbie & Khatiwala, in prep

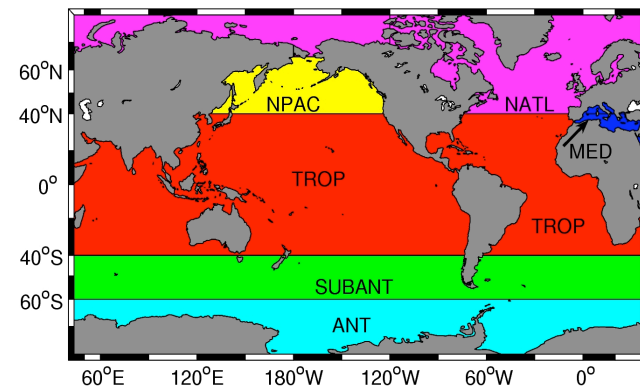
# Volume fraction from regional sources (WOCE clim.)



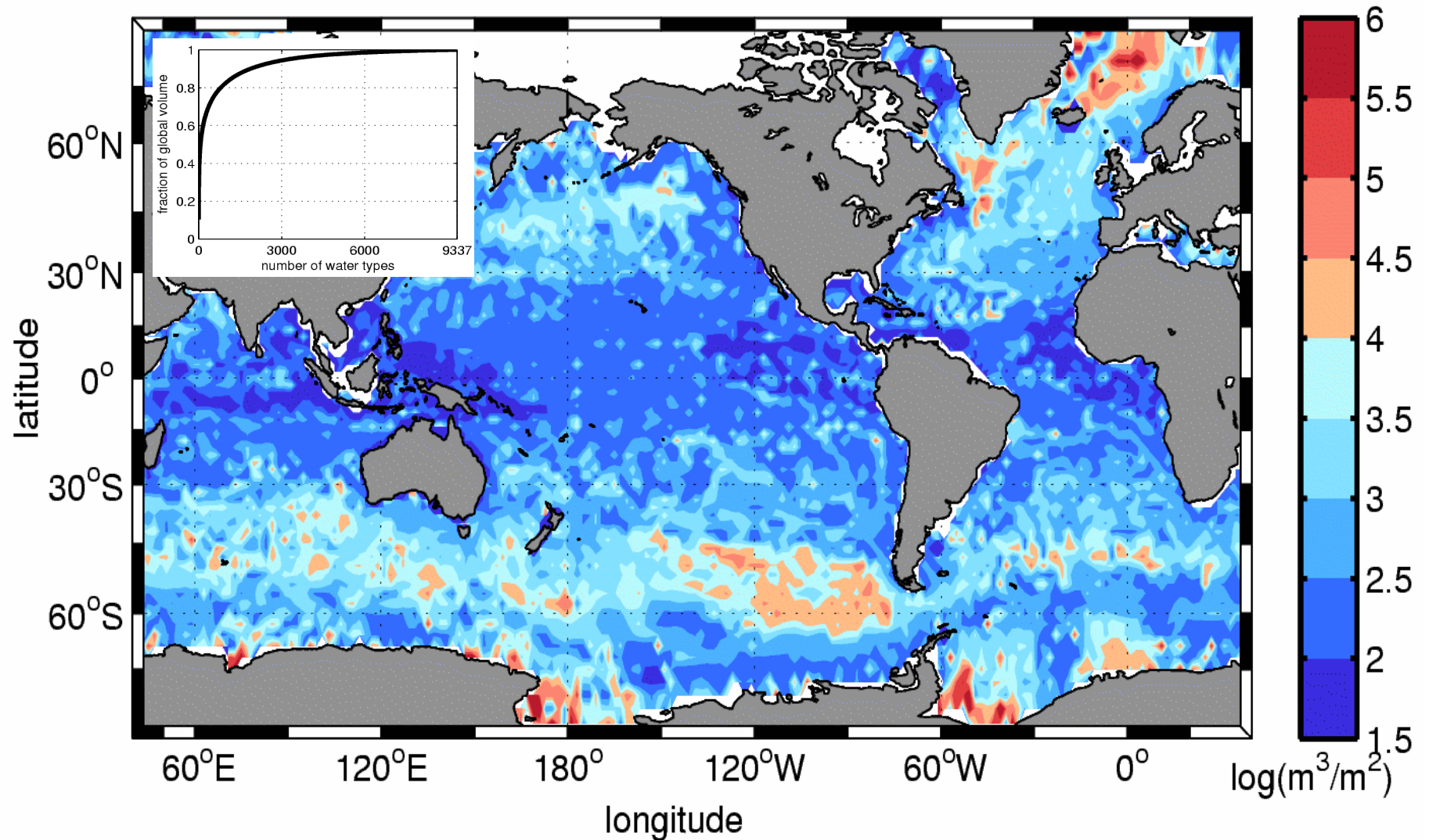
$$g_{natl} = \mathbf{A}^{-1} \mathbf{b}_{natl}$$

Atlantic Ocean section

Gebbie & Huybers, submitted

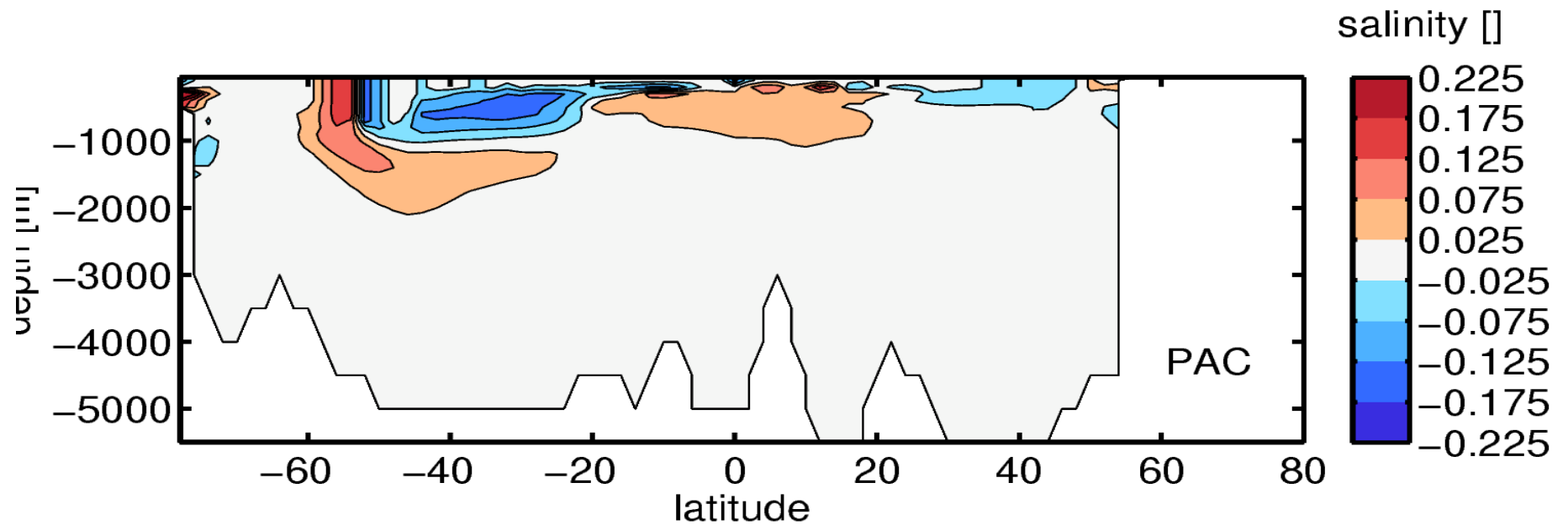
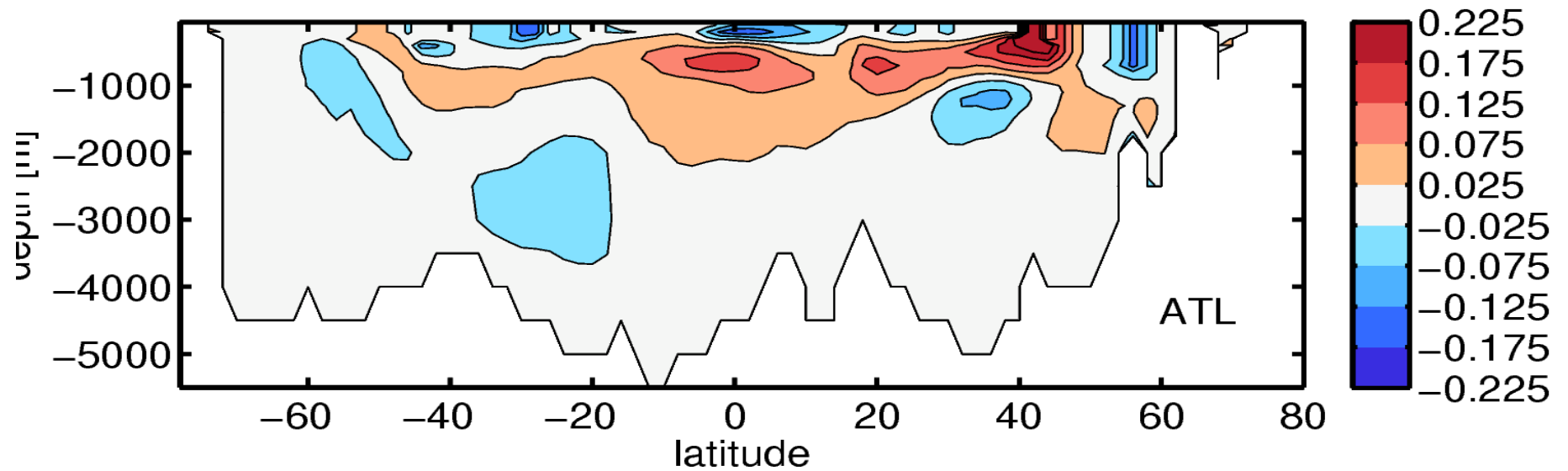


# The surface origin of ocean waters



$$dV/db = \mathbf{A}^{-T} \mathbf{v}, \quad \text{Adjoint Green function}$$

Salinity disequilibrium = steady-state minus observed salinity



## Adjoint-related activities at Harvard

2. Projects with  
the TAF-produced  
MOM4 adjoint model

(Gebbie, Tziperman,  
Lee and Fukumori)

# MOM4\_ad

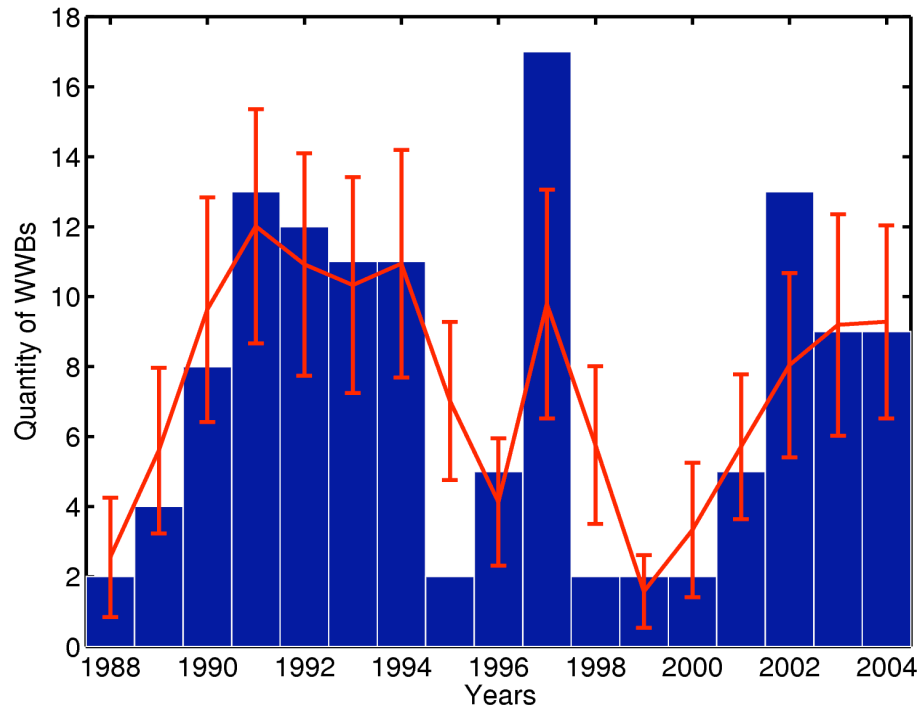


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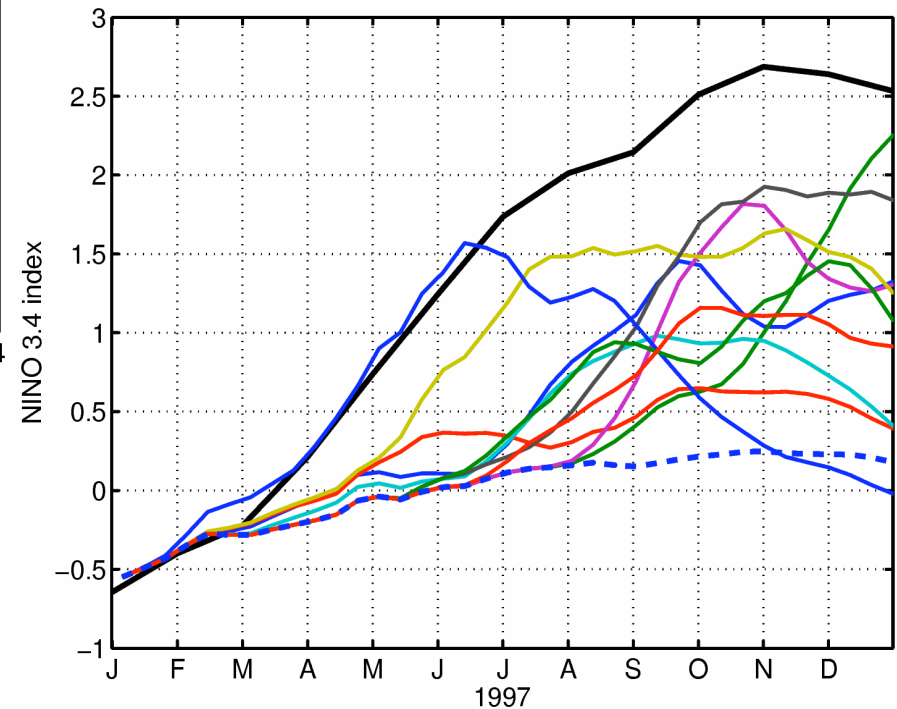
## 2. Projects with the MOM4 forward and adjoint models



Predicted vs. observed WWBs per year

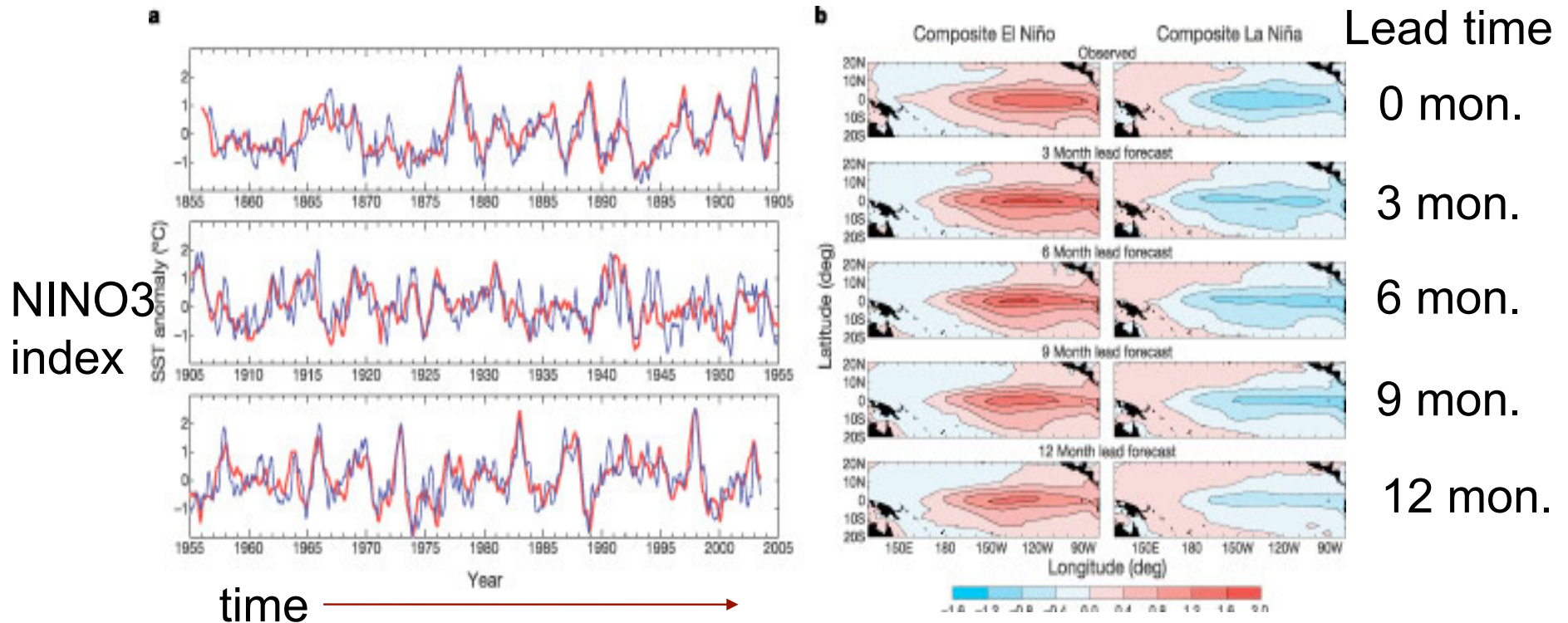
Initialization of the ocean state  
by a simple nudging scheme  
Gebbie, Eisenman, et al. 2007  
Gebbie & Tziperman 2008a,b

A hybrid coupled model using MOM4  
can retrospectively predict the statistics  
of westerly wind bursts (left) and the  
evolution of large El Nino events (below)



Forecasts of the 1997 El Nino versus  
observations (black) of the NINO3.4 index

# State Estimation and ENSO Prediction

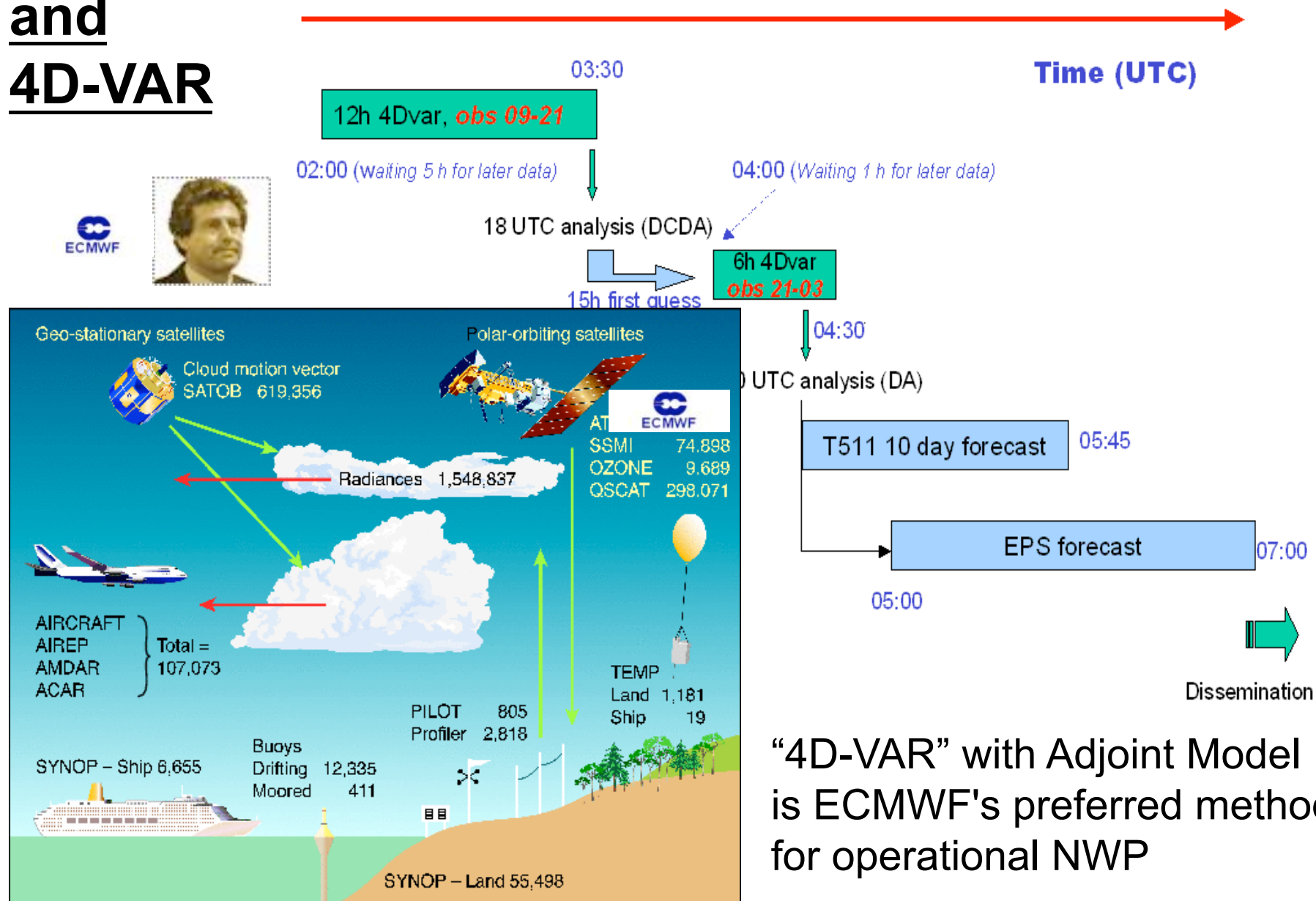


Chen, Zebiak, Cane, et al., 1995, 2005

.An improved data assimilation scheme resulted in significantly improved ENSO forecasts.

**.With a more-realistic model and initialization from a state estimate, expect potential for even greater forecast improvements.**

# ECMWF and 4D-VAR



“4D-VAR” with Adjoint Model  
is ECMWF's preferred method  
for operational NWP

# Adjoint-related activities at Harvard: Conclusions

## 1. Estimating the steady-state circulation

- a. Estimating the spatial pattern of vertical and horizontal diffusion
- b. Is there circulation information in additional tracers? (Nd, C13, 3He)
- c. What is the “age” of the ocean?
- d. Do sediment-core measurements require a different ocean circulation in the past?



## 2. Projects with the TAF-produced MOM4 adjoint model

ENSO prediction: proposed in the recent NOAA call from Climate Program Office

# MOM4\_ad



For any tracer, we now have a relationship between neighboring points

$$C_{ijk} - m_1 C_{i+1,jk} - m_2 C_{i-1,jk} - m_3 C_{i,j+1,k} - m_4 C_{i,j-1,k} - m_5 C_{ij,k+1} - m_6 C_{ij,k-1} = \Delta C_{ijk}$$

Adding Dirichlet boundary conditions, the global tracer field **c** satisfies:

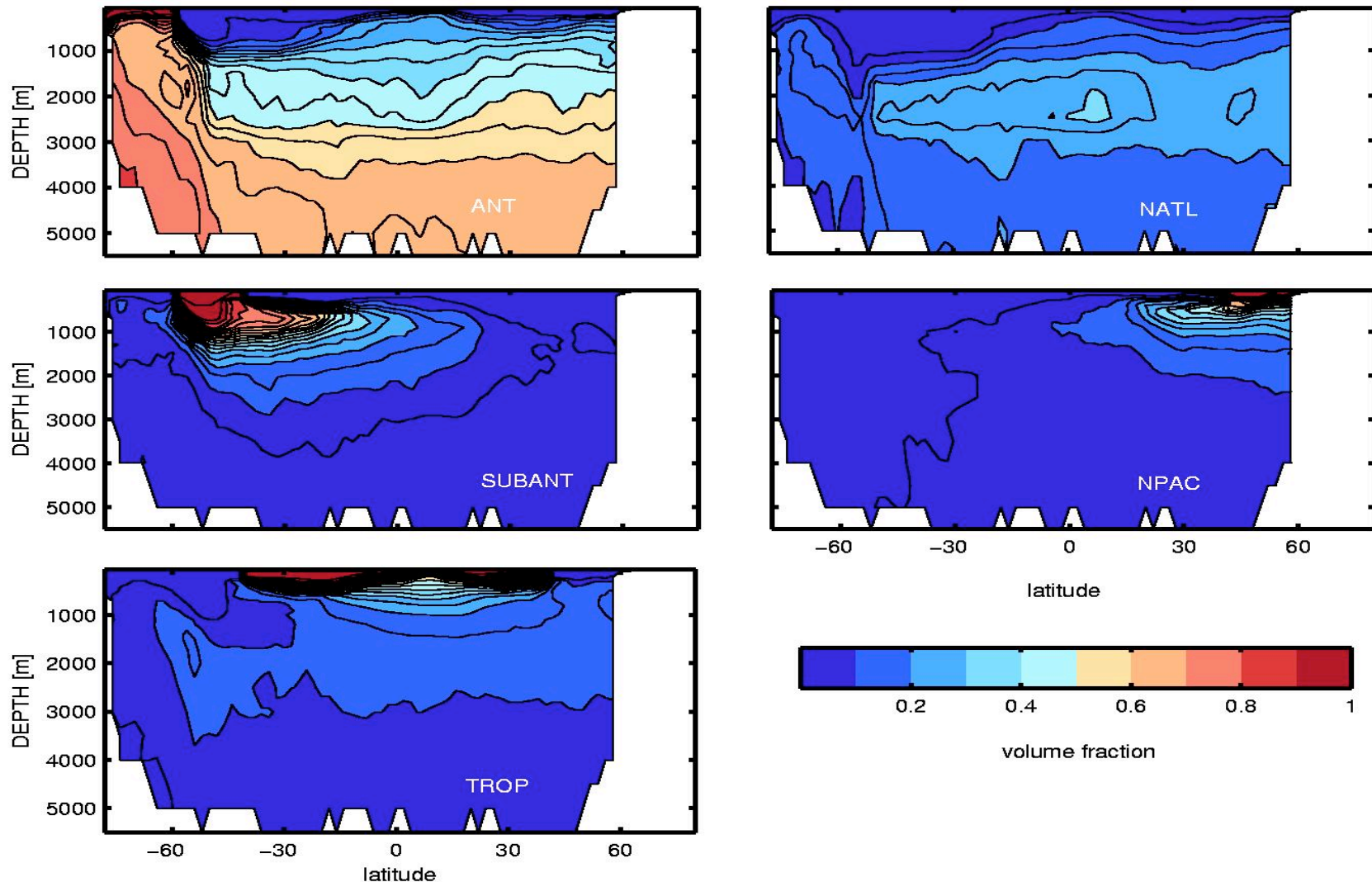
$$C_{ijk} - m_1 C_{i+1,jk} - m_2 C_{i-1,jk} - m_3 C_{i,j+1,k} - m_4 C_{i,j-1,k} - m_5 C_{ij,k+1} - m_6 C_{ij,k-1} = \Delta C_{ijk}$$

$$\mathbf{A}\mathbf{c} = \mathbf{d}$$

(At 2°, **A** has size 180670 x 180670).

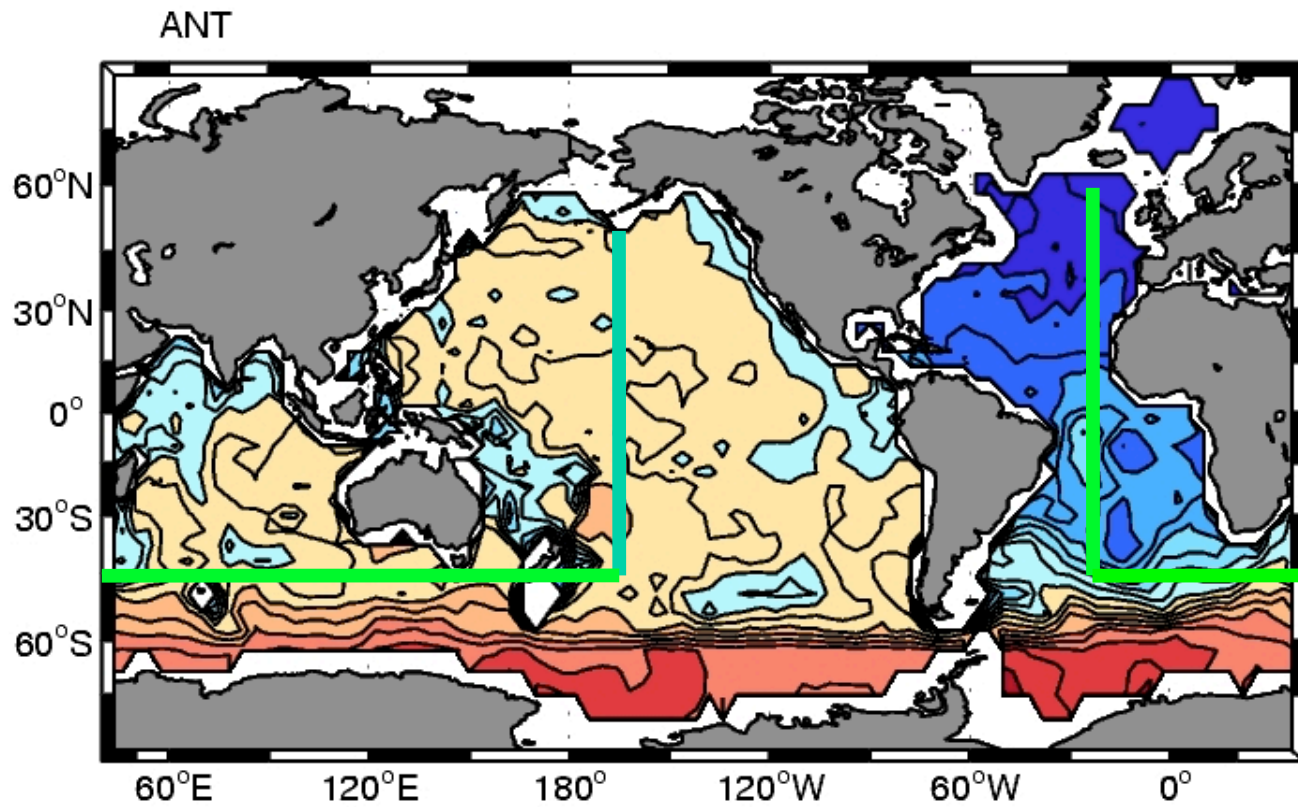
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# Case study: evolution of North Atlantic water in the world ocean

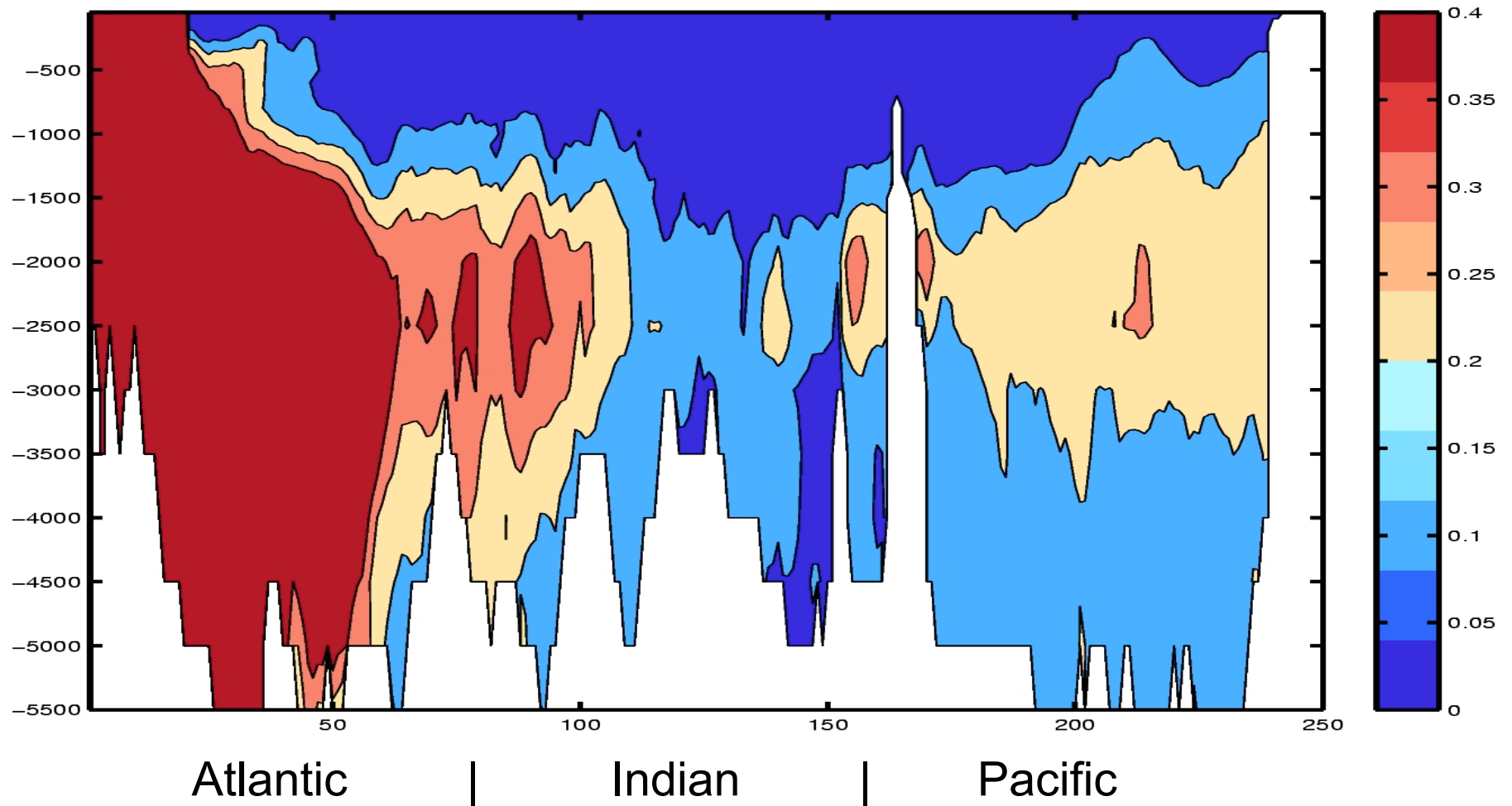


Volume fraction from 5 surface patches  
Latitude-depth sections of the Pacific, 180-150W

## The core path of North Atlantic water through the world ocean



## Case study: evolution of North Atlantic water in the world ocean



The core of North Atlantic water  
throughout the world ocean